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QUALITATIVE SECURITY MODELING IN THE CATEGORY OF FUZZY PROCESS

Jakościowe modele bezpieczeństwa w kategoriach rozmytych

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Abstract

The universality and incredible complexity of the security concept generate enormous problems during the attempts to build a uniform definition of it, which meets formal and substantive criteria. Equally significant problems are implied by the very complicated typology of security, classified in many ways according to various taxonomic criteria. The paper presents the principles and method of using the theory of fuzzy sets by A. L. Zadeh for qualitative analysis of so-called negative security of an entity having certain defense capabilities. The analytical exemplification of the entity's security was carried out based on fuzzy modeling using, for this purpose, the tool that is Mamdani fuzzy controllers (systems). For this purpose, security was defined as a logical function of three variables: threat potential, risk, and defense potential of the entity. The effect of the fuzzy model is a fuzzy security value expressed by a symbolic linguistic variable.

Keywords: defense, fuzzy models, risk, safety, threats.

Streszczenie

Powszechność i niezwykła złożoność pojęcia bezpieczeństwa generuje ogromne problemy przy próbach zbudowania jego jednolitej definicji, spełniającej kryteria formalne i merytoryczne. Równie duże problemy implikuje bardzo skomplikowana typologia bezpieczeństwa, klasyfikowana na wiele sposobów według różnych kryteriów taksonomicznych. W artykule przedstawiono zasady i sposób wykorzystania teorii zbiorów rozmytych A. L. Zadeha do jakościowej analizy bezpieczeństwa podmiotu posiadającego określone zdolności obronne. Analityczna egzemplifikacja bezpieczeństwa podmiotu została przeprowadzona w oparciu o modelowanie rozmyte z wykorzystaniem narzędziowych sterowników E. H. Mamdaniego. W tym celu bezpieczeństwo zostało zdefiniowane jako funkcja logiczna trzech zmiennych: zagrożeń, ryzyka i potencjału obronnego podmiotu. Efektem działania modelu jest rozmyta wartość bezpieczeństwa określona za pomocą zmiennych lingwistycznych.

Słowa kluczowe: obrona, modele rozmyte, ryzyko, bezpieczeństwo, zagrożenia.

Introduction

The universal concept of security, both in theory and practice, functions as an ambiguous and multithreaded category. Although it is crucial for civilizational development and social life, it does not have a specific connotation either in terms of definition or application. According to the philosophical approach, security means the freedom of existence, choice, and action, which guarantees the unrestricted development and self-improvement of the security entity (Pokruszyński, 2013). According to W. Pokruszyński, “the philosophical reflection on security is connected with such philosophical disciplines as (...) axiology, ontology, epistemology, and others”. In a narrower ontological view, security is a kind of being to be cared for and pursued (Ficoń, 2013b). In axiological terms, security is a particular category of the highest value for each human individual and social group. In the praxeological aspect, security is a holistic application that guarantees desired functioning standards in actual reality (Ficoń, 2011). Finally, security is seen as a temporary state and a dynamic process (Kaczmarczyk, 2013).

In social life, extreme, maximum (perfect) security for a given entity is an unattainable ideal state. The real state is always a certain nominal security, containing to a greater or lesser extent factors of threat, that is, not entirely eliminated effects of various threats. So, one can say that the marginal threat is a kind of background for any state of security. It will be reasonable to state that security is a fuzzy, vague, and relative category and always subjective or objective and extraordinarily complex and variable. This is probably why quantitative modeling of security is so challenging and of little practical use. Hence the proposal of a qualitative alternative depicting it using soft, qualitative fuzzy systems theory.

Outline of the fundamentals of fuzzy systems

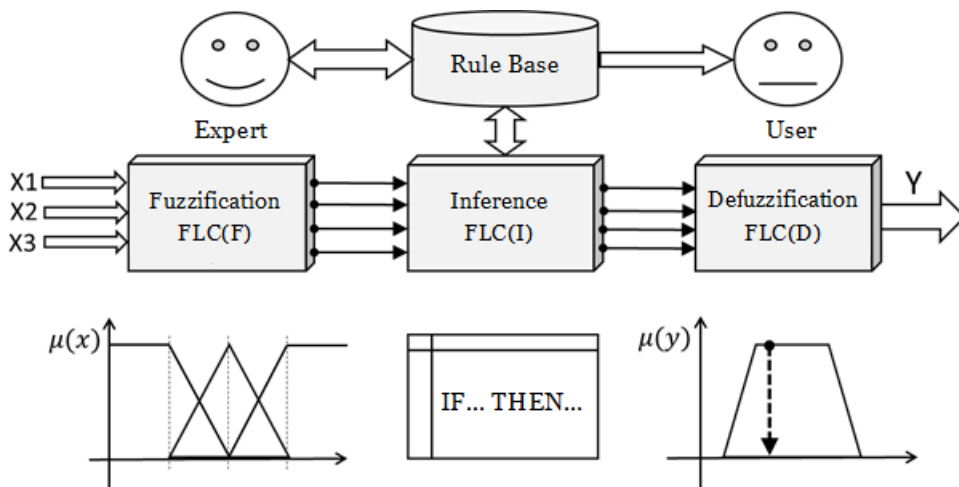
Apart from membership functions, the concept of a fuzzy set is closely related to the concept of a linguistic variable, which will be formally defined as a variable whose values are words or sentences used in natural (or artificial) language (Ficoń, 2012). According to R. Nowicki, “a linguistic variable is a quantity which takes on both numerical and linguistic values,” while “a linguistic value is called a verbal expression of a certain quantity”. (Nowicki, 2009). The following words or sentences illustrate the use of the concepts of linguistic value and linguistic variable.

If in common language we define human height as – “short,” “medium,” “tall,” then height is a linguistic variable, whose values are “short,” “medium,” “tall”. (Łachwa, 2015). Similar linguistic relations are found for such pairings as the speed of a car – “slow,” “medium,” “fast”; the beauty of people – “ugly,” “common,” “pretty,” “beautiful”; the state of household finances – “critical,” “satisfactory” “good”; the internal mood of a person – “depression,” “joy,” “euphoria,” the sense of security – “low,” “medium,” “high,” etc. We can measure some of these variables very precisely with the

help of various instruments, for example, the height of a person, the speed of a car, others unfortunately not – the beauty of people, the inner mood or the state of security. However, we can always subjectively express (evaluate) each of these quantities in the convention of fuzzy sets utilizing a variable and a linguistic value. As noted by P. Kulczycki (2007), “the linguistic approach is the basis of traditional fuzzy control, which is a standard example of fuzzy sets applications”.

The theoretical considerations of L. Zadeh were given a praxeological impetus only by the utilitarian concept of fuzzy logic controllers (FLC) proposed by E.H. Mamdani (1976). Fuzzy controllers implement the so-called fuzzy inference, which can be carried out either based on precise analytical data (for example, measurements) or the basis of vague linguistic variables. However, their operation is always controlled by a specific set of fuzzy logic rules, implemented in the structure of an appropriate knowledge base (Rutkowski, 2006).

Figure 1.
Conceptual diagram of Mamdani fuzzy controller.



Source: own work.

Potential security formulation

The basis of modeling in Fuzzy Set theory’s convention is always a certain logical model of the process (system) under study, taking into account the methodological assumptions made and the necessary parameters and decision variables. In this case, the decision variable sought is the system security function (Q), whose arguments are: the threat potential (X), risk (R), and the response potential (Y), which will be written down in the form of a simplified security formula, (Ficoń, 2007) as:

$$Q = f(X, R, Y) \quad (1)$$

where:

Q – system security function,
 X – system threat potential,
 R – the risk function,
 Y – the entity's response potential.

All arguments of function (1) are complex implicit functions of detailed variables describing each argument. In particular, each of these arguments is a complex function of many other variables and constants, the form of which is difficult to determine analytically and – when operating with a tool of Fuzzy sets using linguistic variables – completely unnecessary.

According to the general concept of the potential security formula (1), we assume that the threat potential X is a function of four aggregated arguments, i.e.:

$$X = f(X_1, X_2, X_3, X_4) \quad (2)$$

where:

X_1 – natural threats,
 X_2 – civilizational threats,
 X_3 – social threats,
 X_4 – other categories of threats.

Each of the above variables $X_i; i = \overline{1,4}$ can be a complex function of other variables and parameters X_{ij} , which we will not deal with in the model being built. Instead, according to Fuzzy theory's assumptions, we will describe them with appropriate linguistic variables and characteristic intervals of variation (term). Similarly, the response potential Y of the operating system, whose security Q has been violated by the threat function X , will be described as a complex function of three arguments:

$$Y = f(Y_1, Y_2, Y_3) \quad (3)$$

where:

Y_1 – the command system potential,
 Y_2 – the executive system potential,
 Y_3 – the master system support potential.

Again, each of the above variables $Y_i; i = \overline{1,3}$ can be a complex function of other variables and parameters Y_{ij} , which we will also not deal with in the Fuzzy model under construction. We will describe the required characteristics with appropriate linguistic variables and fixed variation intervals (term).

The risk function R is expressed by the classical formula of the so-called computational risk as the product of the probability of occurrence of a critical event (threat) and its predicted consequences (Wolanin, 2005):

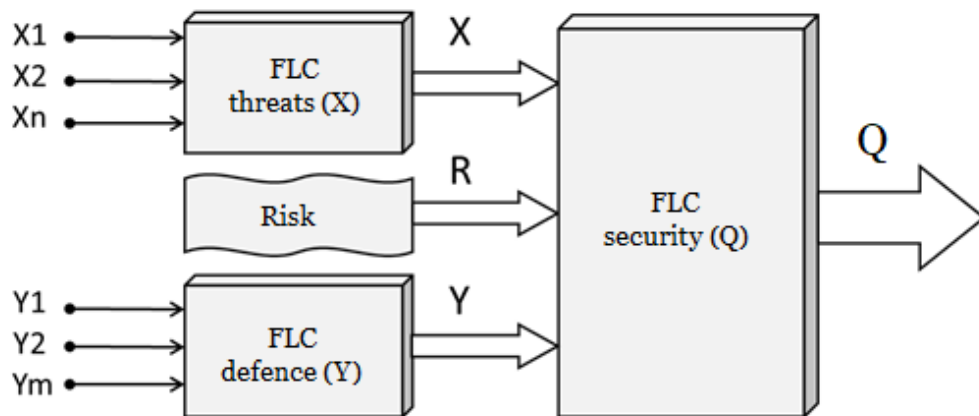
$$R = f(p, \$) \tag{4}$$

where: $\$$ – the predicted impact of a critical event (threat).

We will use the formal apparatus of fuzzy set theory and Mamdani's fuzzy controller models (Rykaczewski, 2006) for multi-criteria estimation of the sought value of the security function Q . For this purpose, we will write the individual functions (2), (3) and (5) in the form of elementary Mamdani's FLCs (Fuzzy Logic Controller) as $FLC(X)$, $FLC(Y)$, and $FLC(Q)$, respectively, which will be directly used to determine the fuzzy values of threat potential X , response potential Y , and the search security function Q . The general concept of using L.A. Zadeh's Fuzzy theory and Mamdani controllers based on its principles to determine the fuzzy value of system security Q is shown in Figure 2.

Figure 2.

Serial-parallel FLC(Q) controller for determining the fuzzy security value Q.



Source: own work.

As a result, we will model the fuzzy system security function Q based on the two-level serial-parallel Mamdani fuzzy controller represented by expression (5):

$$\left. \begin{array}{l} X_1(X_{1i}) \\ X_2(X_{2i}) \\ X_3(X_{3i}) \\ X_4(X_{4i}) \end{array} \right\} FLC(X(X_i)) \\
 \left. \begin{array}{l} Y_1(Y_{1i}) \\ Y_2(Y_{2i}) \\ Y_3(Y_{3i}) \end{array} \right\} FLC(Y(Y_i)) \\
 \left. \begin{array}{l} R(p, S) \\ \\ \\ \end{array} \right\} FLC(Q(X, R, Y)) \tag{5}$$

As can be seen from equation (5), the process of determining the security function using fuzzy modeling $FLC(Q)$ has been decomposed into three separate steps concerning the construction of three standard Mamdani controllers $FLC(X)$, $FLC(Y)$, and $FLC(Q)$. These are classical fuzzy controllers FLC having, in this case, four or three signal inputs and one control output, being the defuzzified parameter sought. To determine the fuzzy value of the system security function Q , a Mamdani controller model will be used using fuzzy linguistic variables, logical conditional sentences, and an expert inference rule base (Wierzchoń, 2009).

Determination of fuzzy value of threat potential

According to the assumptions of the $FLC(X)$ model, the fuzzy value of threat potential X will be determined based on four linguistic variables – natural threats (X_1), civilizational threats (X_2), social threats (X_3), and other threats (X_4) (Ficoń, 2007). We assume that the individual independent variables ($X_1 \div X_4$) can take linguistic values as in Table 1, which are additionally scaled as natural numbers from a conventional variation interval. Due to program instructions’ requirements, all linguistic variables will be represented in plain text notations (without indices) in the following discussion.

Table 1.

Linguistic values and terms of input arguments threat categories – X1, X2, X3, X4.

Threats	Terms of the set $X(X_1, X_2, X_3, X_4)$		
Natural (X1) (0÷10)	Low $X1M \leq 4$	Medium $2 \leq X1S \leq 8$	High $X1D \geq 6$
Civilizational (X2) (0÷10)	Low $X2M \leq 3$	Medium $1 \leq X2S \leq 7$	High $X2D \geq 5$
Social (X3) (0÷10)	Low $X3M \leq 5$	Medium $3 \leq X3S \leq 9$	High $X3D \geq 7$
Other (X4) (0÷10)	Low $X4M \leq 4$	Medium $2 \leq X4S \leq 90$	High $X4D \geq 7$

An analogous set of linguistic variables describing the output fuzzy set X is shown in Table 2.

Table 2.

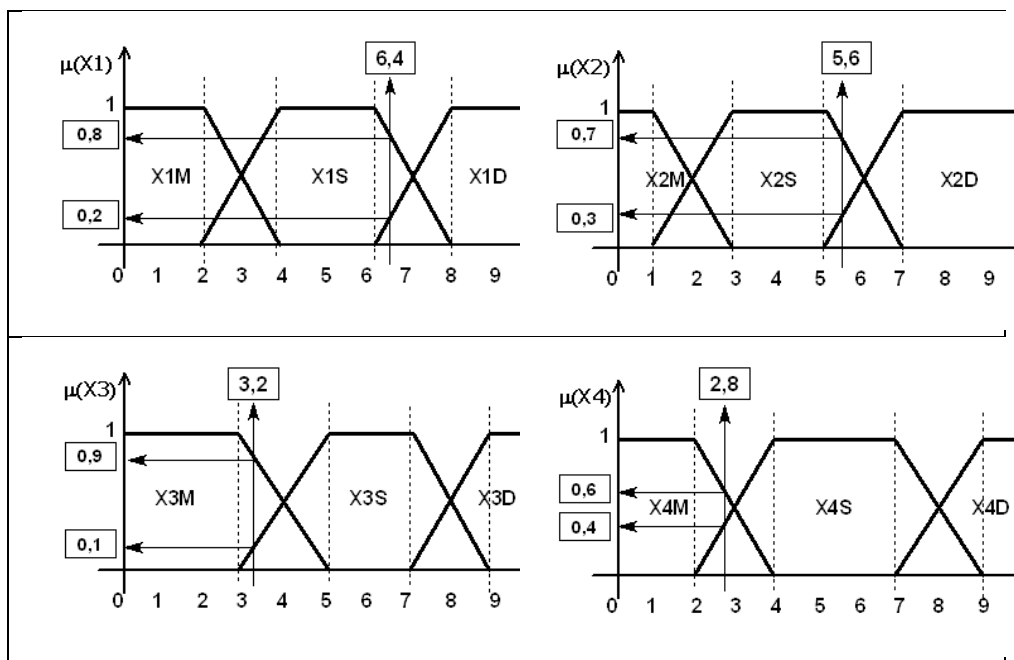
Linguistic values and terms of the output set threat potential – X.

Threat potential – X(0-100)				
Very low	Low	Medium	High	Very high
$YBM \leq 2$	$1 \leq YMA \leq 4$	$3 \leq YSR \leq 7$	$6 \leq YDU \leq 9$	$YBD \geq 8$

A graphical depiction of the linguistic variables X1, X2, X3, and X4, along with the corresponding fuzzy sets X1, X2, X3, and X4 and example membership functions $\mu(X1)$, $\mu(X2)$, $\mu(X3)$, $\mu(X4)$ (Rutkowski, 2006), are shown in Figures 3 and 4.

Figure 3.

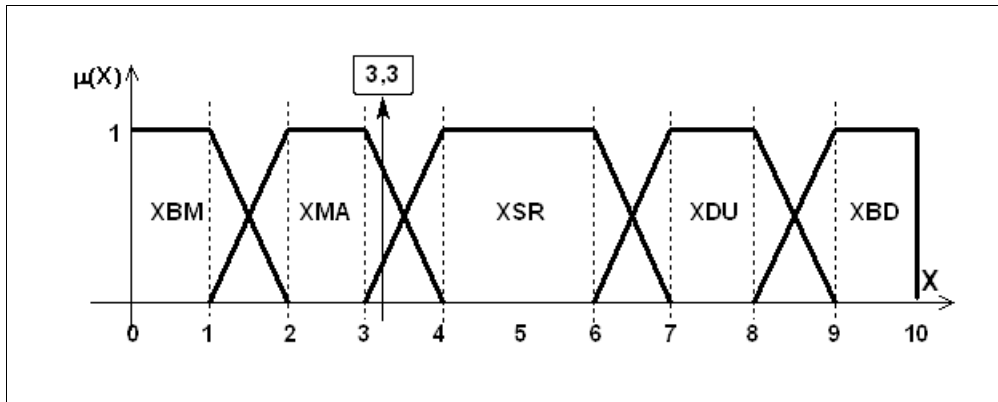
Terms and degrees of membership of fuzzy sets X1, X2, X3, X4.



Source: own work.

Figure 4.

Terms and linguistic value of the fuzzy set potential threats.



Source: own work.

In order to use the designed fuzzy sets X_1 , X_2 , X_3 , and X_4 in practice, we assume that, according to the expert, the linguistic variables X_1 , X_2 , X_3 and, X_4 currently take on the following numerical values:

- $X_1 = 6.4$ – natural threats,
- $X_2 = 5.6$ – civilizational threats,
- $X_3 = 3.2$ – social threats,
- $X_4 = 2.8$ – other threats.

To determine the degrees of membership of these values to the appropriate fuzzy sets – “low” (M), “medium” (S), “high” (D) one should use the analytical form of the appropriate membership function – in this case, we will use the trapezoidal function (Wierchoń, 2009).

Assumed example defuzzified values of the linguistic variable $X_1 = 6.4$; $X_2 = 5.6$; $X_3 = 3.2$; $X_4 = 2.8$ correspond to the following degrees of membership – Table 3.

Table 3.

Degrees of membership of sample values of defuzzified threats to fuzzy sets X_1 , X_2 , X_3 , X_4 .

Fuzzy set	Degrees of membership		
X_1 – natural threats	$\mu_{X_1M}(6.4) = 0$	$\mu_{X_1S}(6.4) = 0.8$	$\mu_{X_1D}(6.4) = 0.2$
X_2 – civilizational threats	$\mu_{X_2M}(5.6) = 0$	$\mu_{X_2S}(5.6) = 0.7$	$\mu_{X_2D}(5.6) = 0.3$
X_3 – social threats	$\mu_{X_3M}(3.2) = 0.9$	$\mu_{X_3S}(3.2) = 0.1$	$\mu_{X_3D}(3.2) = 0$
X_4 – other threats	$\mu_{X_4M}(2.8) = 0.6$	$\mu_{X_4S}(2.8) = 0.4$	$\mu_{X_4D}(2.8) = 0$

The data in Table 3 can be interpreted as follows. For example, the value of “natural threats” $X1 = 6.4$ corresponds to a membership degree of 0.8 to the fuzzy set $X1S$ – “medium threats” and a degree of 0.2 to the fuzzy set $X1D$ – “high threats”. Similarly, the degrees of membership of separated categories of threats are interpreted for the remaining fuzzy sets – $X2, X3, X4$.

To evaluate the fuzzy value of the threat potential (X), we will use an inference scheme based on rules defined by an expert whose technical knowledge has been implemented, for example, by a knowledge engineer in the form of a set of logical rules based on the E.H. Mamdani model. In the case of the classical most popular fuzzy system containing two inputs and one output, the selection of inference rules can be pre-planned using a two-dimensional decision matrix, whose coordinates are the input variables – fuzzy sets, which are the premises of inference, and the elements of the matrix are the designed expert conclusions (Piegat, 2003).

In the case when there are more than two fuzzy variables in the input of the Mamdani model, the corresponding decision table is multidimensional, which does not change the inference mechanism itself. Taking into account the fact that during the design of the logical rule base, only active rules from the total number of rules $3^4 = 81$ are considered, as can be seen from Table 3, the subject of interest of the expert will be a limited set of rules including a total of $2^4 = 16$ different rules, of the following form, as shown in Table 4.

Table 4.
Base of expert logical rules estimating threat potential.

1.	If	$X1=X1S$	and	$X2=X2S$	and	$X3=X3M$	and	$X4=X4M$	then	$X=BM$
2.	If	$X1=X1S$	and	$X2=X2S$	and	$X3=X3M$	and	$X4=X4S$	then	$X=BM$
3.	If	$X1=X1S$	and	$X2=X2S$	and	$X3=X3S$	and	$X4=X4M$	then	$X=MA$
4.	If	$X1=X1S$	and	$X2=X2S$	and	$X3=X3S$	and	$X4=X4S$	then	$X=MA$
5.	If	$X1=X1S$	and	$X2=X2D$	and	$X3=X3M$	and	$X4=X4M$	then	$X=MA$
6.	If	$X1=X1S$	and	$X2=X2D$	and	$X3=X3M$	and	$X4=X4S$	then	$X=SR$
7.	If	$X1=X1S$	and	$X2=X2D$	and	$X3=X3S$	and	$X4=X4M$	then	$X=SR$
8.	If	$X1=X1S$	and	$X2=X2D$	and	$X3=X3S$	and	$X4=X4S$	then	$X=SR$
9.	If	$X1=X1D$	and	$X2=X2S$	and	$X3=X3M$	and	$X4=X4M$	then	$X=SR$
10.	If	$X1=X1D$	and	$X2=X2S$	and	$X3=X3M$	and	$X4=X4S$	then	$X=SR$
11.	If	$X1=X1D$	and	$X2=X2S$	and	$X3=X3S$	and	$X4=X4M$	then	$X=SR$
12.	If	$X1=X1D$	and	$X2=X2S$	and	$X3=X3S$	and	$X4=X4S$	then	$X=DU$
13.	If	$X1=X1D$	and	$X2=X2D$	and	$X3=X3M$	and	$X4=X4M$	then	$X=DU$
14.	If	$X1=X1D$	and	$X2=X2D$	and	$X3=X3M$	and	$X4=X4S$	then	$X=DU$
15.	If	$X1=X1D$	and	$X2=X2D$	and	$X3=X3S$	and	$X4=X4M$	then	$X=BD$
16.	If	$X1=X1D$	and	$X2=X2D$	and	$X3=X3S$	and	$X4=X4S$	then	$X=BD$

According to the general scheme of fuzzy inference in the inference block, we will select logical rules due to their validity. To evaluate the degrees of membership of the individual input variables X1 – “natural threats,” X2 – “civilizational threats,” X3 – “social threats,” X4 – “other threats” in set X – threat potential, it is necessary to check the condition of the truth of the premises, i.e., compliance with the assumptions made. As shown in Table 3, out of the admissible set, including $3^4 = 81$ rules, only $2^4 = 16$ rules, characterized by the premises’ truth satisfy this condition. According to Mamdani’s scheme, we will apply a fuzzy operator of type MIN-MAX to the above active rules. For this purpose, we will determine the minimum from the degrees of membership of the individual premises of each rule, which is expressed by the following system of logical equations (Table 5):

Table 5.

Mechanism of the Mamdani operator on the set of logical rules contained in Table 4.

1.	0.8/X1S	\wedge	0.7/X2S	\wedge	0.9/X3M	\wedge	0.6/X4M	=	0.6/XBM
2.	0.8/X1S	\wedge	0.7/X2S	\wedge	0.9/X3M	\wedge	0.4/X4S	=	0.4/XBM
3.	0.8/X1S	\wedge	0.7/X2S	\wedge	0.1/X3S	\wedge	0.6/X4M	=	0.1/XMA
4.	0.8/X1S	\wedge	0.7/X2S	\wedge	0.1/X3S	\wedge	0.4/X4S	=	0.1/XMA
5.	0.8/X1S	\wedge	0.3/X2D	\wedge	0.9/X3M	\wedge	0.6/X4M	=	0.3/XMA
6.	0.8/X1S	\wedge	0.3/X2D	\wedge	0.1/X3M	\wedge	0.4/X4S	=	0.1/XSR
7.	0.8/X1S	\wedge	0.3/X2D	\wedge	0.1/X3S	\wedge	0.6/X4M	=	0.1/XSR
8.	0.8/X1S	\wedge	0.3/X2D	\wedge	0.1/X3S	\wedge	0.4/X4S	=	0.1/XSR
9.	0.2/X1D	\wedge	0.7/X2S	\wedge	0.9/X3M	\wedge	0.6/X4M	=	0.2/XSR
10.	0.2/X1D	\wedge	0.7/X2S	\wedge	0.9/X3M	\wedge	0.4/X4S	=	0.2/XSR
11.	0.2/X1D	\wedge	0.7/X2S	\wedge	0.1/X3S	\wedge	0.6/X4M	=	0.1/XSR
12.	0.2/X1D	\wedge	0.7/X2S	\wedge	0.1/X3S	\wedge	0.4/X4S	=	0.1/XDU
13.	0.2/X1D	\wedge	0.3/X2D	\wedge	0.9/X3M	\wedge	0.6/X4M	=	0.2/XDU
14.	0.2/X1D	\wedge	0.3/X2D	\wedge	0.9/X3M	\wedge	0.4/X4S	=	0.2/XDU
15.	0.2/X1D	\wedge	0.3/X2D	\wedge	0.1/X3S	\wedge	0.6/X4M	=	0.1/XBD
16.	0.2/X1D	\wedge	0.3/X2D	\wedge	0.1/X3S	\wedge	0.4/X4S	=	0.1/XBD

The values of the linguistic variables of the left side of the system of equations (Table 4) refer to the premises of the set of active logical rules $\{1,2,\dots,16\}$, connected by conjunction operator (AND). According to operator MIN, the right side of this system contains minimal elements. In order to obtain a fuzzy inference result coming from the rules $\{1, 2, \dots, 16\}$, we will apply to the layout (Table 4) the MAX operator, whose operation is illustrated by the following expression:

$$0.6/BM \vee 0.4/BM \vee 0.1/MA \vee 0.1/MA \vee 0.3/MA \vee 0.1/SR \vee 0.1/SR \vee 0.1/SR \vee 0.2/SR \vee 0.2/SR \vee 0.1/SR \vee 0.1/DU \vee 0.2/DU \vee 0.2/DU \vee 0.1/BD \vee 0.1/BD = 0.6/XBM \vee 0.3/XMA \vee 0.2/XSR \vee 0.2/XDU \vee 0.1/XBD \tag{6}$$

The fuzzy inference results occurring on the right side of equation (6) determine the partial execution potential Y's value resulting from the expert's opinion. In practice, operating with five fuzzy sets and the membership function described in them is not very communicative, so this information in the defuzzification block is subject to defuzzification, that is, transformation to a numerical linguistic value. According to the adopted methodology, we will perform the defuzzification operation using the popular center of gravity (COG) method. For computational reasons, we will use its simplified discrete version of the form (7), which we will formally write as:

$$X = \frac{\sum_{i=1}^I (w_i \times \Omega_i)}{\sum_{i=1}^I w_i} \tag{7}$$

$$\frac{0.6 \times \Omega(\text{BM}) + 0.3 \times \Omega(\text{MA}) + 0.2 \times \Omega(\text{SR}) + 0.2 \times \Omega(\text{DU}) + 0.1 \times \Omega(\text{BD})}{0.6 + 0.3 + 0.2 + 0.2 + 0.1}$$

where:

- $\Omega(\text{XBM}) = \{0,0,1,2\} = 0.75$
- $\Omega(\text{XMA}) = \{1,2,3,4\} = 2.5$
- $\Omega(\text{XSR}) = \{3,4,6,7\} = 5$
- $\Omega(\text{XDU}) = \{6,7,8,9\} = 7.5$
- $\Omega(\text{XBD}) = \{8,9,10,10\} = 9.25$

$$X = \frac{0.6 \times 0.75 + 0.3 \times 2.5 + 0.2 \times 5 + 0.2 \times 7.5 + 0.1 \times 9.25}{0.6 + 0.3 + 0.2 + 0.2 + 0.1} = 3.3 \tag{8}$$

The linguistic fuzzy value of the threat potential FLC(X) on the assumed scale of variation (0-10) is based on the assumptions made (Table 3) and the expert knowledge base (Table 4) is X = 3.3, which is the solution to the stage task.

Determination of the fuzzy response potential value

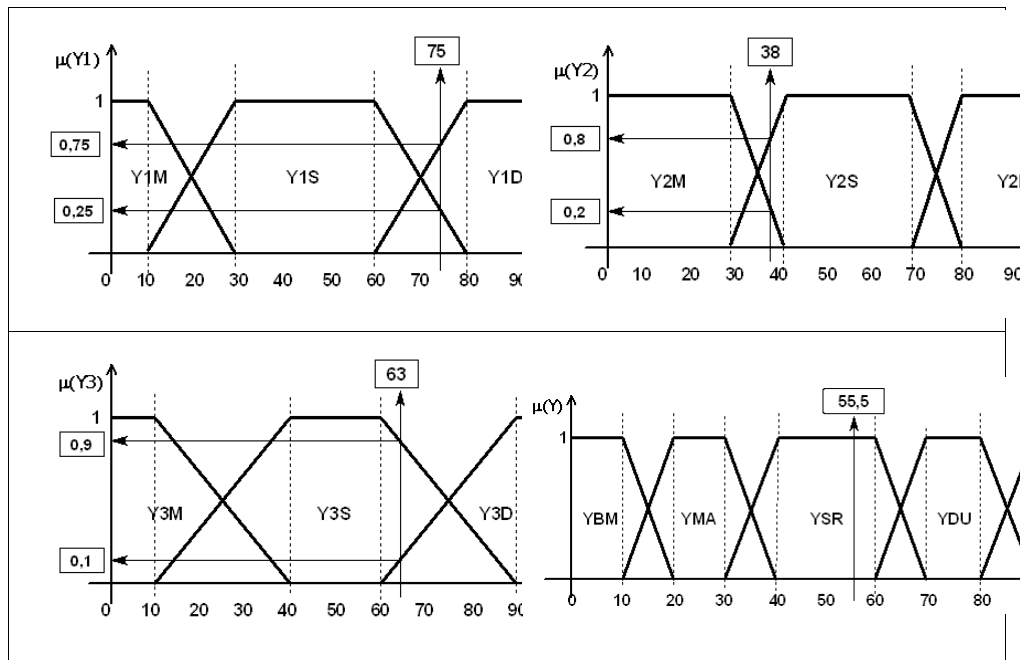
According to the assumptions of FLC(Y) model, the fuzzy value of response potential Y will be determined based on three linguistic variables characterizing the potential of the command system (Y1), the executive system (Y2), and the master support system (Y3). We assume that the individual independent variables (Y1, Y2, Y3) can take on linguistic values as in Table 6, which have additionally been scaled as natural numbers from a conventional range of variation.

Table 6.
Linguistic values and terms of input arguments Y_1, Y_2, Y_3

Potential	Fuzzy set terms Y_1, Y_2, Y_3		
Command (Y_1) (0÷100)	Low $Y_{1M} \leq 30$	Medium $10 \leq Y_{1S} \leq 80$	High $Y_{1D} \geq 60$
Executive (Y_2) (0÷100)	Low $Y_{2M} \leq 40$	Medium $30 \leq Y_{2S} \leq 80$	High $Y_{2D} \geq 70$
Support (Y_3) (0÷100)	Low $Y_{3M} \leq 40$	Medium $10 \leq Y_{3S} \leq 90$	High $Y_{3D} \geq 60$

A graphical depiction of the linguistic variables $Y_1, Y_2,$ and Y_3 and the corresponding fuzzy sets $Y_1, Y_2,$ and Y_3 , along with example membership functions $\mu(Y_1), \mu(Y_2)$ and $\mu(Y_3)$, are shown in Figures 2 and 3.

Figure 5.
Terms and degrees of membership of fuzzy sets Y_1, Y_2, Y_3, Y .



Source: own work.

Table 7.

Linguistic values and terms of the output variable Y – response potential.

Response potential – Y				
Very low YBM ≤20	Low 10 ≤ YMA ≤40	Medium 30 ≤ YSR ≤70	High 60 ≤ YDU ≤90	Very high YBD ≥80

To use the designed fuzzy sets Y1, Y2, Y3, in practice, we assume that, according to the expert, the linguistic variables Y1, Y2, Y3 currently take the following numerical values:

- Y1 = 75 – command potential,
- Y2 = 38 – executive potential,
- Y3 = 63 – support potential.

To determine the degrees of membership of these values to the corresponding fuzzy sets: “low” (YiM), “medium” (YiS), and “high” (YiD), it is necessary to use the analytical form of the appropriate membership function; also, in this case, we will use the trapezoidal function. The assumed defuzzified values of the linguistic variable Y1 = 75, Y2 = 38, X3 = 63 correspond to the following degrees of membership (Table 8):

Table 8.

Degrees of membership of the sample defuzzified values to fuzzy sets Y1, Y2, Y3.

Fuzzy sets	Degrees of membership		
Y1 – command potential	$\mu_{Y1M}(75) = 0$	$\mu_{Y1S}(75) = 0.25$	$\mu_{Y1D}(75) = 0.75$
Y2 – executive potential	$\mu_{Y2M}(38) = 0.2$	$\mu_{Y2S}(38) = 0.8$	$\mu_{Y2D}(38) = 0$
Y3 – support potential	$\mu_{Y3M}(43) = 0$	$\mu_{Y3S}(43) = 0.9$	$\mu_{Y3D}(43) = 0.1$

The data in Table 8 can be interpreted as follows. For example, the values of “command potential” Y1 = 75 correspond to membership in degree 0.25 to the fuzzy set Y1S – “medium potential” and in degree 0.75 to the fuzzy set Y1D – “high potential”. Similarly, the degrees of membership of separated potential categories are interpreted for the remaining fuzzy sets Y2, Y3.

To evaluate the fuzzy value of the response potential (Y), we will use an inference scheme based on rules defined by an expert whose technical expertise has been implemented, for example, by a knowledge engineer in the form of a set of logical rules based on the Mamdani model. A complete rule base for three variables Y1, Y2, Y3 taking three possible linguistic values YiM, YiS, YiD should contain a total of $3^3 = 27$ different decision options. According to the Mamdani model’s assumptions, only so-called

active rules are required for closer analyses, which means that the expert's set of permissible decisions of the expert will contain $2^3 = 8$ possible decision permutations.

Table 9.

Expert logical rule base estimating response potential

1.	If	Y1=Y1S	and	Y2=Y2M	and	Y3=Y3S	then	Y=YBM
2.	If	Y1=Y1S	and	Y2=Y2M	and	Y3=Y3D	then	Y=YMA
3.	If	Y1=Y1S	and	Y2=Y2S	and	Y3=Y3S	then	Y=YMA
4.	If	Y1=Y1S	and	Y2=Y2S	and	Y3=Y3D	then	Y=YSR
5.	If	Y1=Y1D	and	Y2=Y2M	and	Y3=Y3S	then	Y=YSR
6.	If	Y1=Y1D	and	Y2=Y2M	and	Y3=Y3D	then	Y=YSR
7.	If	Y1=Y1D	and	Y2=Y2S	and	Y3=Y3S	then	Y=YDU
8.	If	Y1=Y1D	and	Y2=Y2S	and	Y3=Y3D	then	Y=YBD

According to Mamdani's scheme, we will apply a fuzzy operator of type MIN-MAX to the above active rules (Table 9). For this purpose, we will determine the minimum from the degrees of membership of the individual premises of each rule, which is expressed by the following system of logical equations (Table 10).

Table 10.

Operation mechanism of the Mamdani operator on the set of logical rules contained in Table 9.

1.	0.25/Y1S	\wedge	0.2/Y2M	\wedge	0.9/Y3S	=	0.2/YBM
2.	0.25/Y1S	\wedge	0.2/Y2M	\wedge	0.1/Y3D	=	0.1/YMA
3.	0.25/Y1S	\wedge	0.8/Y2S	\wedge	0.9/Y3S	=	0.25/YMA
4.	0.25/Y1S	\wedge	0.8/Y2S	\wedge	0.1/Y3D	=	0.1/YSR
5.	0.75/Y1D	\wedge	0.2/Y2M	\wedge	0.9/Y3S	=	0.2/YSR
6.	0.75/Y1D	\wedge	0.2/Y2M	\wedge	0.1/Y3D	=	0.1/YSR
7.	0.75/Y1D	\wedge	0.8/Y2S	\wedge	0.9/Y3S	=	0.75/YDU
8.	0.75/Y1D	\wedge	0.8/Y2S	\wedge	0.1/Y3D	=	0.1/YBD

The values of the linguistic variables of the left side of the system of equations (Table 10) refer to the premises of the set of active logical rules $\{1, 2, \dots, 8\}$, connected by the conjunction operator (AND). According to operator MIN, the right side of this system contains minimal elements. In order to obtain the fuzzy inference result from the rules $\{1, 2, \dots, 8\}$, we will apply to the layout (Table 10) the MAX operator, whose operation is illustrated by the following expression:

$$0.2/BM \vee 0.1/MA \vee 0.25/MA \vee 0.1/SR \vee 0.2/SR \vee 0.1/SR \vee 0.75/DU \vee 0.1/BD = \\ = 0.2/YBM \vee 0.25/YMA \vee 0.2/YSR \vee 0.75/YDU \vee 0.1/YBD \quad (9)$$

The fuzzy inference results occurring on the right side of equation (---) determine the partial response potential Y's value resulting from the expert's opinion. In practice, operating with five fuzzy sets and the membership function described on them is not very communicative, so this information in the defuzzification block is subject to defuzzification, that is, transformation to a numerical linguistic value. According to the adopted methodology, we will perform the defuzzification operation using the simplified center of gravity (COG) method, which we will formally write as:

$$\frac{0.2 \times \Omega(BM) + 0.25 \times \Omega(MA) + 0.2 \times \Omega(SR) + 0.75 \times \Omega(DU) + 0.1 \times \Omega(BD)}{0.2 + 0.25 + 0.2 + 0.75 + 0.1}$$

where:

$$\begin{aligned} \Omega(YBM) &= \{0,0,10,20\} = 7.5 \\ \Omega(YMA) &= \{10,20,30,40\} = 25 \\ \Omega(YSR) &= \{30,40,60,70\} = 50 \\ \Omega(YDU) &= \{60,70,80,90\} = 75 \\ \Omega(YBD) &= \{80,90,100,100\} = 92.5 \end{aligned}$$

$$Y = \frac{0.2 \times 7.5 + 0.25 \times 25 + 0.2 \times 50 + 0.75 \times 75 + 0.1 \times 92.5}{0.2 + 0.25 + 0.2 + 0.75 + 0.1} = 55.5 \quad (11)$$

The linguistic fuzzy value of the response potential FLC(Y) in the adopted variation scale (0-100) based on the adopted assumptions (Table 9) and the expert knowledge base (Table 10) is Y = 55.5, which is the solution of the stage task.

Determination of fuzzy security value

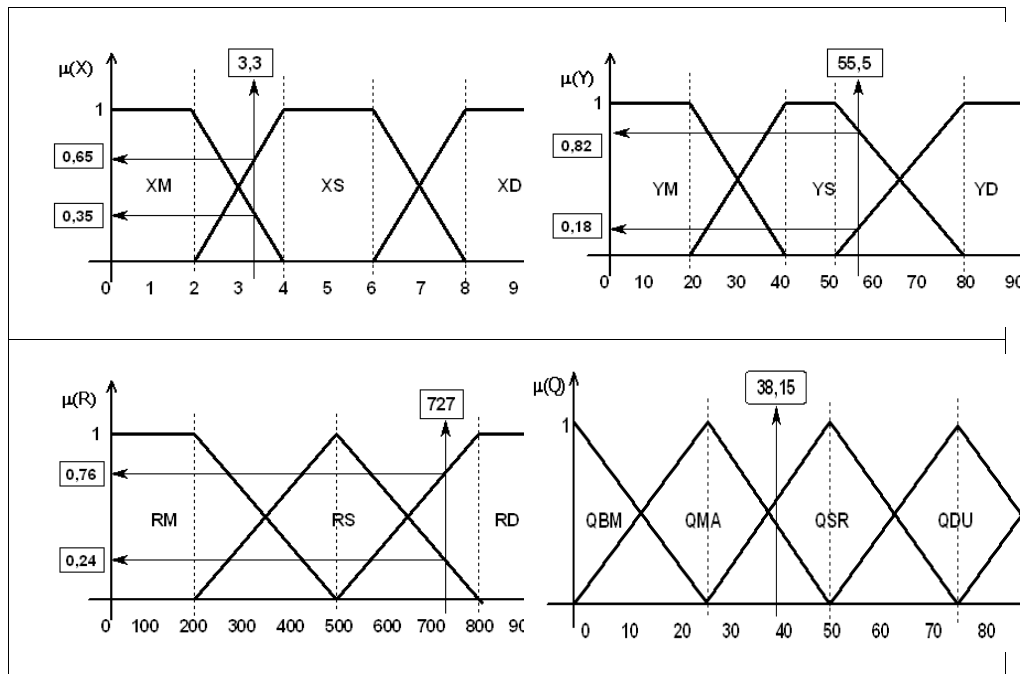
According to the concept of potential security formula PFB and the assumptions of the FLC(Q) model, the fuzzy security value Q will be determined based on three linguistic variables, including: threat potential (X), response potential (Y), and threat transformation risk I. The first two linguistic variables – threat potential (X) and response potential (Y), have been determined analytically as fuzzy variables using special FLC(X) and FLC(Y) models. We will arbitrarily take the numerical value of risk directly from the so-called computational definition of risk as a certain real number (Ficoń, 2013a). We will consider all the FLC(Q) controller input variables in the fuzzy sets category, as linguistic variables, whose general characteristics are shown in Table 11.

Table 11.
Linguistic values and terms of input arguments X, Y, R of the FLC(Q) controller.

Input variables	X,Y,R fuzzy set terms		
Threat potential (X) (0÷10)	Low $XM \leq 4$	Medium $2 \leq XS \leq 8$	High $XD \geq 6$
Response potential (Y) (0÷100)	Low $YM \leq 40$	Medium $30 \leq YS \leq 80$	High $YD \geq 70$
Risk I (0÷1,000)	Low $RM \leq 350$	Medium $250 \leq RS \leq 750$	High $RD \geq 650$

A graphical depiction of the linguistic variables Y1, Y2, and Y3 and the corresponding fuzzy sets Y1, Y2, and Y3 along with example membership functions $\mu(Y1)$, $\mu(Y2)$, and $\mu(Y3)$ are shown in Figures 2 and 3.

Figure 5.
Terms and degrees of membership of fuzzy sets X, Y, R, Q.



Source: own work.

Table 12.

Linguistic values and terms of the output variable Q – Security.

Q – Security				
Very low YBM ≤ 25	Low 0 ≤ YMA ≤ 50	Medium 25 ≤ YSR ≤ 75	High 50 ≤ YDU ≤ 100	Very high YBD ≥ 75

To use the designed fuzzy sets X, Y, and R in practice, we assume that, according to the expert’s opinion, the linguistic variables X, Y, and R currently take the following numerical values:

- X = 3.3 – threat potential,
- Y = 55.5 – response potential,
- R = 727 – the risk of threats.

To determine the degrees of membership of these values to the corresponding fuzzy sets: M – “low”, S – “medium”, Y – “high”, it is necessary to use the analytical form of the appropriate membership function – in this case, we will use the trapezoidal function and the triangular function. The assumed defuzzified values of the linguistic variable X = 3.3, Y = 55.5, R = 727, correspond to the following degrees of membership (Table 13).

Table 13.

Degrees of membership of the sample defuzzified values to fuzzy sets X, Y, R.

Fuzzy sets	Degrees of membership		
X – threat potential	$\mu_{XM}(3.3) = 0.35$	$\mu_{XS}(3.3) = 0.65$	$\mu_{XD}(3.3) = 0$
Y – response potential	$\mu_{YM}(55.5) = 0$	$\mu_{YS}(55.5) = 0.82$	$\mu_{YD}(55.5) = 0.18$
R – risk of threats	$\mu_{RM}(727) = 0$	$\mu_{RS}(727) = 0.24$	$\mu_{RD}(727) = 0.76$

The data in Table 13 can be interpreted as follows. For example, the value “risk” R = 727 corresponds to membership in degree 0.24 to the fuzzy set RS – “medium risk” and in degree 0.76 to the fuzzy set RD – “high risk”. The degrees of membership of the potential categories for the remaining fuzzy sets X and Y are interpreted similarly.

To evaluate the fuzzy security value Q, we will use an inference scheme based on rules defined by an expert whose technical knowledge has been implemented, for example, by a knowledge engineer in the form of a set of logical rules based on the Mamdani model. A complete rule base for three variables X, Y, and R taking three possible linguistic values: low – M, medium – S, high – D should contain a total of $3^3 = 27$ different decision options. According to the Mamdani model’s assumptions, only so-called active rules are required for closer analyses, which means that the expert’s set

of permissible decisions will contain $2^3 = 8$ possible decision permutations, the specification of which is provided in Table 14.

Table 14.

The base of expert logical rules estimating the state of security.

1.	If	X = XM	and	Y = YS	and	R = RS	then	Q = QMA
2.	If	X = XM	and	Y = YS	and	R = RD	then	Q = QBM
3.	If	X = XM	and	Y = YD	and	R = RS	then	Q = QDU
4.	If	X = XM	and	Y = YD	and	R = RD	then	Q = QSR
5.	If	X = XS	and	Y = YS	and	R = RS	then	Q = QSR
6.	If	X = XS	and	Y = YS	and	R = RD	then	Q = QSR
7.	If	X = XS	and	Y = YD	and	R = RS	then	Q = QSR
8.	If	X = XS	and	Y = YD	and	R = RD	then	Q = QMA

According to Mamdani’s scheme, we will apply a fuzzy operator of type MIN-MAX to the above active rules (Table 14). For this purpose, we will determine the minimum from the degrees of membership of the individual premises of each rule, which is expressed by the following system of logical equations:

Table 15.

Mechanism of Mamdani operator operation on the set of logical rules contained in Table 14.

1.	0.35/XM	\wedge	0.82/YS	\wedge	0.24/RS	=	0.25/QMA
2.	0.35/XM	\wedge	0.82/YS	\wedge	0.76/RD	=	0.35/QBM
3.	0.35/XM	\wedge	0.18/YD	\wedge	0.24/RS	=	0.18/QDU
4.	0.35/XM	\wedge	0.18/YD	\wedge	0.76/RD	=	0.18/QSR
5.	0.65/XS	\wedge	0.82/YS	\wedge	0.24/RS	=	0.24/QSR
6.	0.65/XS	\wedge	0.82/YS	\wedge	0.76/RD	=	0.65/QSR
7.	0.65/XS	\wedge	0.18/YD	\wedge	0.24/RS	=	0.18/QSR
8.	0.65/XS	\wedge	0.18/YD	\wedge	0.76/RD	=	0.18/QMA

The values of the linguistic variables of the left side of the system of equations (Table 15) refer to the premises of the set of active logical rules {1,2, ..., 8}, connected by the conjunction operator (AND). According to the operator MIN, the right side of this system contains minimal elements. To obtain a fuzzy inference result coming from the rules {1, 2, ..., 8}, we will apply to the layout (Table 15) the MAX operator, whose operation is illustrated by the following expression:

$$0.25/QMA \vee 0.35/QBM \vee 0.18/QDU \vee 0.18/QSR \vee 0.24/QSR \vee 0.65/QSR \vee 0.18/QSR \vee 0.18/QMA = = 0.25/QMA \vee 0.35/QBM \vee 0.18/QDU \vee 0.65/QSR \quad (12)$$

The fuzzy inference results occurring on the right side of equation (12) determine the partial executive potential Y's value resulting from the expert's opinion. In practice, the operation of four fuzzy sets and the membership function described on them is not very communicative, so this information in the defuzzification block is subject to defuzzification, i.e., transformation to a numerical linguistic value. Following our methodology, we will perform the defuzzification operation utilizing the center of gravity (COG) method, using its simplified discrete version (Rutkowski, 2006), which we will formally write as:

$$\frac{0.25 \times \Omega(QMA) + 0.35 \times \Omega(QBM) + 0.18 \times \Omega(QDU) + 0.65 \times \Omega(QSR)}{0.25 + 0.35 + 0.18 + 0.65}$$

where:

$$\begin{aligned} \Omega(QMA) &= \{0,25,50\} &= 25 \\ \Omega(QBM) &= \{0,0,20\} &= 6.6 \\ \Omega(QDU) &= \{50,75,100\} &= 75 \\ \Omega(QSR) &= \{25,50,75\} &= 50 \end{aligned}$$

$$Q = \frac{0.25 \times 25 + 0.35 \times 6.6 + 0.18 \times 75 + 0.65 \times 50}{1.43} = 38.15 \quad (14)$$

The linguistic fuzzy value of the security function FLC(Q) on the conventional scale of variation (0-100) determined from the assumptions made (Table 13) and the expert knowledge base (Table 14) is Q = 38.15, which corresponds to the fuzzy security magnitude in the QSR scope – medium security.

Conclusion

The conducted research has shown the great usefulness of the theory of A. L. Zadeh's fuzzy sets for the systemic analysis of negative security presented in a three-element logical model. As a research tool, E.H. Mamdani fuzzy controllers' method was used, whose computational efficiency, despite grouping in one model as many as four cascading controllers, turned out to be adequate for research needs. Fuzzy sets and Mamdani's tool models illustrate the fuzzy nature of security, its high complexity, and great processual variability.

One disadvantage of the presented approach is the relatively tedious and laborious process of determining particular linguistic variables conditioning the Mamdani fuzzy controller's operation. This procedure can be significantly improved by using well-known software standards of the MATLAB-Simulink package dedicated to fuzzy systems. The specialist Fuzzy Logic Toolbox (FLT) application extends the MATLAB programming environment with tools to design fuzzy logic-based systems. The FLT's beneficial graphical interfaces guide the user through the steps of a fuzzy design and

inference system, essentially making the rather complex methods of fuzzy set theory more useful and accessible.

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